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SIMULATION OF HEAT AND MASS TRANSFER PROCESSES AND A SYSTEM APPROACH

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The paper presents the basic concepts of a system approach to thermal processes and heat-exchanging apparatuses. The principle of groups that is applied to the analysis of thermal systems and the search for systemic, integrative properties is considered. Integral and local thermal and "mechanical" resistances are one of the forms of the groups. Dimensionless complexes of similarity theory are shown to be one of the forms of the groups at whose critical values integrative properties are manifested.

A system approach is a broadly scientific, methodological, and philosophical concept that consists in studying any objects (material and ideal) as systems. The definition of the concept "system" is the concern of the works of many authors. A review of their investigations is made, for example, in [1]. We will call a system any object consisting of subsystems and possessing the properties of interaction and emergence. In our case of consideration of thermal systems these two properties are most interesting, since only their presence allows one to open up the new and make discoveries. Interaction is the presence of links and correlations between subsystems (SS) that are represented by any properties, qualities, specific features, parts, etc. of the object-system (S) under investigation. Emergence is the ability to produce systemic, integrative properties (IP) that are inherent in the whole, i.e., in the system, rather than in its individual parts, i.e., SS, beyond the links, beyond the structure. The latter is also one of the subsystems. The concept "system" is inseparably associated with the concepts of integrity and uniqueness of the initial system and multiplicity of its models, hierarchy, nonadditivity, equivalence, integral and local, continuous and discrete, etc.

The models of an initial physical system are represented by its thermal scheme, i.e., a thermal model and its mathematical models, since some of the subsystems of the initial system are not taken into account in constructing thermal schemes and their mathematical models (MM). The initial system is unique, since rejection, neglect, or addition of any of its properties, i.e., SS, can alter (more often decrease and more rarely increase) the quality and quantity of the possible integrative properties. Thus, the principle of uniqueness of systems and the principle of simulation of physical objects are in dialectical contradiction, which can be overcome by the principle of groups. The latter consists in the following: it is not necessary that all the subsystems produce integrative properties at the same place and at a certain time; the integrative properties occur at certain (they are frequently called "critical") numerical values of the SS groups. Examples of such groups are the critical numbers Re_{cr}, Ra_{cr}, M, and Pr known from the theory of motion and heat transfer, stability criteria, and grid numbers in computational hydroaeromechanics and thermal physics [2-4]. The dimensionless integral complexes of the theory of similarity and dimensionality [5-10] represent only some of the dimensionless and dimensional groups of subsystems that serve as criteria of integrativity (CI) at certain values of them. A great number of forms of the criteria, numbers, and complexes of similarity theory are described in detail in [10], which, moreover, contains very interesting and enlightening, from the methodological viewpoint, information on scientists in whose honor numbers, criteria, complexes, and groups carrying generalized information about physical processes (systems) were, are being, and will be named.

The integrative properties of thermal systems depend not only on the complexes of similarity theory, but also on the critical values of other dimensional and dimensionless groups. Below we will speak about the ways of

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finding the forms and values of the dimensionless groups. The method of self-similar variables, procedures for constructing the forms of local and integral groups by means of integral transformations, and well-known analytical solutions of problems (mathematical models of systems) of motion and heat transfer are described in [5, 11-18].

Let us consider two problems.

1. The problem of "pure" heat conduction makes it possible to obtain integral and local values of similarity criteria, thermal resistances, and local criteria, i.e., grid numbers.

2. The problem of motion and heat transfer with free convection in a large volume makes it possible to obtain integral and local values of groups and "mechanical" resistances. After discretization of continuous media, from thermal and "mechanical" resistances we can obtain analogs of integral complexes, i.e., grid numbers [4], local groups of subsystems for elementary (controlled according to [19]) volumes [20-22]. In [23] it is shown that similarity criteria must depend on directions.

Let a mathematical model (MM), which in our case is the thermal system under investigation, be called a "problem." To construct a mathematical model, we must know causal relations and must employ conservation laws, equations of state, equations of the stressed state, and all the uniqueness equations that connect the system (in our case the MM) with the outer medium. We consider the thermoaerohydrodynamics problems proceeding from a phenomenological approach.

The heat conduction equation in differential form is

$$\sum_{i=1}^{3} \frac{\partial}{\partial x_i}\left(\lambda \frac{\partial T}{\partial x_i}\right) - c_V \frac{\partial T}{\partial \tau} + \sum_{i=1}^{3} u_i c_V \frac{\partial T}{\partial x_i} + q_V = 0 . \tag{1}$$

The boundary conditions have the form

$$\text{of the I kind} \quad T_{sur} = f_1 (x_i, \tau) , \tag{2}$$

$$\text{of the II kind} \quad -\lambda \partial T/\partial n|_{sur} = q_{sur} (x_i, \tau, T) , \tag{3}$$

$$\text{of the III kind} \quad -\lambda \partial T/\partial n|_{sur} = \alpha_{sur} (T_{med} - T_{sur}) , \tag{4}$$

$$\text{of the IV kind} \quad T_{sur1} = T_{sur2} + \Delta T_{cont} ; \; -\lambda_1 \partial T/\partial n|_{sur1} = -\lambda_2 \partial T/\partial n|_{sur2} + q_{cont} , \tag{5}$$

$$T (x_i, 0) = f_2 (x_i) , \quad x_i - x_1, x_2, x_3 . \tag{6}$$

From MM (1)-(6) we derive the following integral groups, numbers, and integral similarity criteria: $\text{Bi} = \alpha_{li}$ $\text{Nu} = \alpha_{liq} L/\lambda_{liq}$, $\text{Fo} = a\tau/L^2$, $\text{Pe} = uL/a$, $\text{Ki} = q_{sur} L/(\lambda \Delta T)$, $\text{Po} = q_V L^2/(\lambda \Delta T)$.

In deriving the criteria, we usually assume all the thermophysical characteristics to be constant, and we assign rather indeterminate L for bodies of complex form, constant scales, and constant values of λ, c_V, u, α, q_{sur}, q_V, $q_{V\alpha} = \alpha_V (T_{med} - T)$, i.e., similarity theory is constructed on the linearization of nonlinear* quantities and the homogenization of variable quantities entering into the uniqueness conditions. Moreover, for bodies of complex form the assignment of determining dimensions is rather arbitrary. Constant scales in are also selected arbitrarily in each particular case.

Let us consider algebraic equations (AE) for elementary volumes ΔV whose dimensions are prescribed, because a grid region is prescribed by calculators. Thus the transition from a system with distributed parameters (SDP) to a system with lumped parameters (SLP) is made.

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* In heat engineering all the quantities entering into the conditions of uniqueness and depending on T are called nonlinearities.

We will write Eq. (1) in an "energy" balance form suitable for both internal elementary volumes and those lying near the surface (in essence, this representation is equivalent to including boundary conditions in algebraic equations):

$$Q_\lambda + Q_\tau + Q_u + Q_{\alpha\,\mathrm{sur}} + Q_{\alpha V} + Q_{q\,\mathrm{sur}} + Q_{qV} = 0\,. \tag{7}$$

This form can be used to derive integral similarity criteria [6], whereas we use it for deriving local complexes-groups. For a node of a grid, for elementary volumes the algebraic equation (the equation of elementary balances [20]) will have the following form if we assume that $Q = \Delta T/R$ or $Q = \Lambda \cdot \Delta T$:

$$\sum_{i=1}^{m} \frac{T_{i,n} - T_{0,n}}{R_{\lambda i}} + \frac{T_{0,n-1} - T_{0,n}}{R_\tau} + \frac{T_{\mathrm{med},n} - T_{\mathrm{sur},n}}{R_\alpha} +$$

$$+ \sum_{j=1}^{3} \frac{T_{j,n} - T_{0,n}}{R_{uj}} + q_{\mathrm{sur},n}\Delta S + q_{V,n}\Delta V = 0\,, \tag{8}$$

where $n = 0, 1, \ldots, 6$ are the numbers of the adjacent nodes; 0 is the central node. Equation (8) can be written for both the implicit and explicit scheme of the grid method (the method of finite differences). In approximating a differential equation (DE) it was assumed that the heat fluxes were proportional to the temperature gradients and drops, i.e., a linear approximation of functions of T was actually carried out. The values of the thermal resistances $R = 1/\Lambda$ are coefficients of the algebraic equations. These coefficients are groups of subsystems entering into the uniqueness conditions of the mathematical model. For volumetric problems $m = 6$; $j = 3$ for convective terms; n is the number of the time step. In the general case $h_1 \neq h_2 \neq \ldots \neq h_6$, $\Delta S_{xy} \neq \Delta S_{xz} \neq \Delta S_{yz}$. For a cube $\Delta V = h^3$, $\Delta S = h^2$; $h = 2l$. Equation (8) can be written for the following schemes of disposition of the nodes: "nodes in the interior" or "nodes at the corners" [21, 22]. Systems of algebraic equations can be solved on any type of computer (analog, digital, hybrid, i.e., AC, DC, and HC) [14].

The thermal resistances R or conductivities Λ are coefficients of the algebraic equations, and using them it is possible to judge approximately quantitatively and qualitatively the quality of the solution (stability [19] and variability [14] for explicit and implicit schemes). The law of coefficients [19] yields the conditions of the variability and stability that give the critical values of the groups, i.e., the criteria of integrativity.

For a rectangular coordinate system the *local* thermal resistances-groups have the form

$$R_{\lambda x1} = \frac{h_{x1}}{\lambda_{x1}\Delta S_{yz}}\,;\quad R_{\lambda x2} = \frac{h_{x2}}{\lambda_{x2}\Delta S_{yz}}\,;\ \ldots\,;$$

$$R_{\lambda z2} = \frac{h_{z2}}{\lambda_{z2}\Delta S_{xy}}\,;\quad R_\tau = \frac{\delta\tau}{c_V\Delta V}\,;\quad R_{ux} = \frac{h_{x1}}{u_{x1}c_V\Delta V}\,;\ \ldots\,;$$

$$R_{u3} = \frac{h_{z1}}{u_z c_V \Delta V}\,;\quad R_{\alpha\,\mathrm{sur}} = \frac{1}{\alpha_{\mathrm{sur}}\Delta S}\,;$$

$$R_{\alpha V} = \frac{1}{\alpha_V \Delta V}\,;\quad h_1 = h_{x1}\,;\quad h_2 = h_{x2}\,;\quad \ldots\,;\quad h_6 = h_{z2}\,. \tag{9}$$

For uniform grids ($h_{x1} = h_{x2} = \ldots = h_{z2} = h$) expressions (9) are simplified:

$$R_\lambda = \frac{h}{\lambda h^2}\,;\quad R_\tau = \frac{\delta\tau}{c_V h^3}\,;\quad R_u = \frac{h}{u c_V h^3}\,;\quad R_{\alpha\,\mathrm{sur}} = \frac{1}{\alpha_{\mathrm{sur}} h^2}\,;\quad R_{\alpha V} = \frac{1}{\alpha_V h^3}\,. \tag{10}$$

The parameters of the grids, i.e., the thermal resistances-groups, differ by a constant value depending on whether the procedure of calculation of the AE coefficients is "nodes at the corners" or "nodes in the interior" [21,

22]. Note that $\alpha_{sur}\Delta S = \alpha_V \Delta V$, i.e., $R_{\alpha\,sur} = R_{\alpha V}$, because differential equations are written for points, W/m^3, whereas algebraic equations are written for elementary volumes, W; $q_{sur}\Delta S = q_V \Delta V$.

It should be emphasized that local thermal resistances (9), i.e., the AE coefficients-groups for nonuniform grids and variable and nonlinear conditions of uniqueness are not identical: they depend on coordinates, time, and temperature. The integral complexes, numbers, and similarity criteria are likewise groups that consist of integral thermal resistances, but they contain not local, but rather integral values of all the subsystems of the thermal system. For example, $\mathrm{Bi} = \alpha L/\lambda = R_{\lambda L\Sigma}/R_{\alpha\Sigma}$, where $R_{\lambda\Sigma}$ and $R_{\alpha\Sigma}$, taken in one direction, refer to the same area $S = 1$ normal to the direction of the heat fluxes Q_λ and Q_α, whereas the grid number $\Delta\mathrm{Bi} = \alpha_x h_x/\lambda_x$ consists of the local values $R_{\lambda\,loc} = h_x/(\lambda_x \Delta S_{yz})$ and $R_{\lambda\,loc} = 1/(\alpha_x \Delta S_{yz})$. The local values α_x, λ_x, ΔS_{yz}, and h_x should be selected taking account of the directions of the heat fluxes.

The integral and local numbers consist of two "types" of quantities: 1) those entering into just the uniqueness conditions and 2) those entering into the uniqueness conditions and the unknown functions T and ΔT (the energy terms in Eq. (8)). The second quantities are the generalized variables, i.e., the desired functions. The first ones are the generalized arguments. One of the ways of finding the groups is to decrease the number of arguments by reducing them to groups (methods of similarity theory, the method of self-similar variables).

Let us consider the relationships of local thermal resistances, i.e., not simply the relationships between the elementary subsystems (SS), but the relationships between the SS groups entering into the thermal resistances. We may speak of the hierarchy of groups beginning with the elementary subsystems, i.e., the groups of the zero order (λ, c, ρ, etc.), and ending with the fullest groups represented by explicit mathematical models, i.e., the solutions of the problems containing all (!) the elementary subsystems in quite definite relationships. These relationships (relations, interrelations, interdependences) generate the desired integrative properties (IP). The integrative properties are generated by the relationships between the quantities entering into the uniqueness conditions and the relationships between the "types of energies," and some of these must be determined after solution, for example, Q_λ, Q_τ, Q_u, and Q_α.

The numerical solution depends on the values of the coefficients of algebraic equations, and the behavior of the solution can already be foreseen from the relationships between these coefficients, i.e., the thermal resistances. This is the manner of production of "grid numbers," stability criteria for implicit finite-difference schemes, variability criteria for implicit schemes, and the law of coefficients [19], which reflects the conservation laws for grid nodes.

The integral criteria of the type $\mathrm{Bi} = R_{\lambda\Sigma}/R_{\alpha\Sigma}$ are constructed proceeding from certain directions of heat fluxes in one-dimensional problems. At the same time, if we want to speak of heat fluxes Q in different directions, we must take the corresponding lengths, areas, λ, α, etc. For example, for rectangular fins with $\delta << H$, to obtain the integral Bi, one cannot take $R_{\lambda\Sigma} = H/(\lambda S)$, where H is the fin height, S is the cross section of the fin, and $R_{\alpha\,lat\,\Sigma} = 1/(\alpha_{lat} \Pi H)$, where Π is the perimeter of the fin ($\Pi = 2(\delta + B)$), B is the fin width, and $R_{\lambda\Sigma H}$ reflects the rate of heat flux along (!) the fin and $R_{\alpha\,lat\,\Sigma}$ across (!) the fin. One cannot construct the integral Bi proceeding from R_λ and R_α in different (!) directions. But the ratio $R_{\lambda\Sigma}/R_{\alpha\,lat\,\Sigma}$ is an important complex that determines the integrative properties of the fin. The group

$$mH = \sqrt{\alpha \Pi H/(\lambda S/H)} = \sqrt{R_{\alpha\Sigma}/R_{\lambda\Sigma H}} \tag{11}$$

was not obtained in the same manner as the integral criteria of the type Bi, Fo, etc. in similarity theory [6-10]. This group is the result of consideration of the analytical solution [24]. Thus, analytical solutions have one other advantage over numerical solutions: they carry information about the composition of groups on whose values the integrative properties of thermal systems depend.

A further example of such a group is the solution [24]

$$\frac{T - T_{med}}{T_p - T_{med}} = \frac{\mathrm{ch}\, mH \left(1 - \frac{x}{H}\right)}{\mathrm{ch}\, mH}, \tag{12}$$

where T_f is the temperature of the fin base.

One of the groups obtained for the thermal system "fin" is the well-known coefficient of finning efficiency E:

$$E = \frac{\text{th } mH}{\sqrt{m}} = \frac{H^2 (\text{th } \sqrt{R_{\lambda H}/R_{\alpha\Sigma}})}{\sqrt{R_{\lambda H}/R_{\alpha\Sigma}}}.$$

As is seen, the integral thermal resistances $R_{\lambda\Sigma H}$ and $R_{\alpha\Sigma}$ form the basis for this group. But the form E can be obtained only by carrying out an analytical investigation that in our case is one of the operations of system analysis. Thus, not only does the mH group reflect the *internal* relations of our thermal system, but also the group (12) and the group E. Groups (11), (12), and E were obtained for the simplest thermal systems, but they suggested still another way for obtaining the qualitative composition (form) of the group.

Analytical solutions reflect internal interrelations and interdependences (interaction and emergences) that represent the basic specific feature of systems.

Let us consider the relationships between the local thermal resistances:

$$\frac{R_\tau}{R_\lambda} = \frac{\delta\tau\lambda\Delta S}{c_V \Delta V h} = \Delta\text{Fo}. \tag{13}$$

Only for the case $\Delta S = h^2$ and $\Delta V = h^3$ do we have

$$\Delta\text{Fo} = \frac{\lambda\delta\tau}{c_V h^2} = \frac{a\delta\tau}{h^2}. \tag{14}$$

The grid number defined by Eq. (14) gives the value of the stability criterion for explicit schemes and the criterion of variability for implicit schemes of the finite-difference method (1/2; 1/4; 1/6). These numbers were found for problems with boundary conditions of the first kind, i.e., for inner grid nodes. The procedure for deriving the Neumann stability condition [4, 19] and analogous conditions of stability and variability is elementary. From the conservation conditions it follows that

$$\frac{\Delta T_1}{R_{\lambda 1}} + \frac{\Delta T_2}{R_{\lambda 2}} + \frac{\Delta T_\tau}{R_\tau} = 0 \quad \text{(one-dimensional problem)}. \tag{15}$$

Assuming that $\Delta T_1 = \Delta T_2 = \Delta T_\tau = \Delta T$, we obtain the condition necessary for an elementary volume (cube):

$$\frac{2}{R_\lambda} = \frac{1}{R_\tau}; \quad \frac{a\delta\tau}{h^2} = \frac{1}{2}. \tag{16}$$

In a similar manner we can obtain other values of this group, i.e., of the integrativity criterion for thermal systems (systems with lumped parameters), for two- or three-dimensional grid regions, for nodes on surfaces with prescribed values α, q_{sur}, q_v, u, etc. For example, for movable sources

$$\frac{R_\lambda}{R_u} = \frac{huc_V\Delta V}{\lambda\Delta\text{Sh}} = \Delta\text{Pe}. \tag{17}$$

The Peclet grid number for a cube will be $\Delta\text{Pe} = uh/a$.

Similarly we obtain $R_\tau/R_u = u\partial\tau/h = \Delta\text{Sh}$ (the local Strouhal number [9]). It is clear that in the equation of motion and heat transfer the term with the heat capacity $c_V\partial T/\partial\tau$ and the convective term were considered. The equation

$$Q_\tau + Q_u + Q_a = 0,$$

yields a relation that takes account of three heat fluxes, and the quantity $\delta\tau$ will depend on c_V, u, ΔV (i.e., h), and α [15-17]. The ratio $R_u/R_\alpha = \alpha/(c_V u) = \Delta\mathrm{St}$, which is the Stanton grid number. For crossflow heat exchanging apparatuses-recuperators such numbers, groups, criteria of variability are obtained in [15-17]. These relations explain the physical meaning of the law of Scarborough coefficients presented in [19].

The local groups

$$\Delta\mathrm{Ki} = \frac{q_{\mathrm{sur}} h}{\lambda \Delta T_\lambda}; \quad \Delta\mathrm{Po} = \frac{q_V h^2}{\lambda \Delta T_\lambda} \tag{18}$$

can be obtained by prescribing ΔT_λ. The quantities reciprocal to these dimensionless numbers are used as dimensionless temperatures:

$$\Theta = \frac{1}{\Delta\mathrm{Ki}} = \frac{(T - T_{\mathrm{nom}})\lambda}{q_{\mathrm{sur}} h}; \quad \Theta = \frac{(T - T_{\mathrm{nom}})\lambda}{q_V h^2} = \frac{1}{\Delta\mathrm{Po}}. \tag{19}$$

The integral dimensionless temperatures are

$$\Theta = \frac{1}{\mathrm{Ki}}; \quad \Theta = \frac{1}{\mathrm{Po}}, \quad \text{where} \quad \Delta T_\lambda = T - T_{\mathrm{nom}}, \tag{20}$$

T_{nom} is the prescribed scale temperature.

The criterion of stability or variability must be determined not from Eq. (16), but from an equation that takes account of heat transfer with the wall. At prescribed values of α the intervals $\delta\tau$ and h must be determined from the algebraic equation for a node, i.e., from

$$\sum_{i=1}^{m} \frac{\Delta T_{\lambda i}}{R_{\lambda i}} + \frac{\Delta T_\tau}{R_\tau} + \frac{\Delta T_u}{R_u} + \frac{\Delta T_{\mathrm{med}}}{R_\alpha} = 0, \tag{21}$$

where m depends on the position of the node with respect to the surface.

Thus, the choice of $\delta\tau$ and h is determined by the velocity u and α when a simplified conjugate problem is solved [15-17].

As is seen, the criteria of integrativity are obtained from conservation laws on the condition of equality of the gradients and the drops ΔT. Actually, in each particular case $\Delta T_\lambda \neq \Delta T_\tau \neq \Delta T_u \neq \Delta T_\alpha$, and therefore *system* phenomena where the quantity $\delta\tau$ can be prescribed larger for explicit schemes than according to condition (16) seem to be "paradoxes." For implicit schemes there are cases where vibrations should must occur at $\delta\tau_{\mathrm{cr}}$, but they are absent. This is explained by the fact that the quantities $\delta\tau_{\mathrm{cr}}$ are derived assuming equality of ΔT_i in Q_i (see Eq. (8)).

Now we will consider a more complex thermal system, namely, a mathematical model of motion and convective heat transfer in a large volume:

$$\frac{\partial u_x}{\partial x} + \frac{\partial u_y}{\partial y} = 0, \tag{22}$$

$$u_x \frac{\partial u_x}{\partial x} + u_y \frac{\partial u_x}{\partial y} = \nu \frac{\partial^2 u_x}{\partial y^2} + \beta g (T - T_\infty), \tag{23}$$

$$u_x \frac{\partial T}{\partial x} + u_y \frac{\partial T}{\partial y} = a \frac{\partial^2 T}{\partial y^2}, \tag{24}$$

$$y = 0 \quad u_x = 0, \quad u_y = 0, \quad T = T_w; \tag{25}$$

$$y = \infty \quad u_x = 0, \quad T = T_\infty. \tag{26}$$

All the simplifications and assumptions adopted in the formulation of the thermal and mathematical model are described in detail in the literature; however, even such a simplified system has a sufficient number of subsystems and can form a basis for deriving the forms of groups that are called the "mechanical" resistances.* Now, we will write the system of differential equations (22)-(26) in the form of algebraic equations. In this case the mass conservation (continuity) equation does not give us grid numbers. Equation of motion (23) can be written in the form of a "balance" equation for the forces A:

$$A_{ux} + A_{uy} = A_{vy1} + A_{vy2} + A_\beta. \tag{27}$$

If we assume that the forces (as above the heat fluxes) are approximated in the form of the expression $A = \Delta u / R^{\text{mech}}$, then from Eq. (27) we obtain such an algebraic equation for a node (for the volume ΔV):

$$\frac{\Delta U_x}{R^{\text{mech}}_{uxx}} + \frac{\Delta U_x}{R^{\text{mech}}_{uxy}} = \frac{\Delta U_{x1}}{R^{\text{mech}}_{vy1}} + \frac{\Delta U_{x2}}{R^{\text{mech}}_{vy2}} + \beta g \Delta T \Delta V = 0. \tag{28}$$

In the thermodynamics of irreversible processes the following form of representation for the generalized fluxes I_i and forces X_K is used: $I_i = \sum_K L_{ik} X_K$. Thus, $R^{\text{mech}} = 1/L_{ik}$, i.e., $1/R^{\text{mech}}$ are the kinetic coefficients, Δu are the generalized forces, and A are the fluxes. Formally and actually, Eq. (28) is the result of linearization of the function u.

The "mechanical" resistances R^{mech} are coefficients at the gradients Δu. In this case directions play an even greater part than in searching for R^{th}:

$$R^{\text{mech}}_{uxx} = \frac{h_x}{u_x \Delta V}; \quad R^{\text{mech}}_{uxy} = \frac{h_y}{u_y \Delta V}; \quad R^{\text{mech}}_{vy1} = \frac{h_{y1}}{\nu \Delta S_{xz}};$$

$$R^{\text{mech}}_{vy2} = \frac{h_{y2}}{\nu \Delta S_{xz}}; \quad A_\beta = \beta g \Delta T \Delta V. \tag{29}$$

Expressions (29) for the groups called "mechanical" resistances are derived similarly to expressions for thermal resistances. Since in the convective terms the unknown quantities T and u enter into R or R^{mech}, the differential and algebraic equations are highly nonlinear.

If u is assigned in the energy equation, then it is an unknown function in the motion equation. Therefore, in numerical solutions of motion equations it is necessary to take into account nonlinearities according to an iterative or noniterative scheme of allowance for nonlinearities [14].

In the expression for the lifting force A_β the drop ΔT is prescribed from a solution of heat conduction equation (24). In this way a solution is obtained for the joint problem of motion and heat transfer, in whose mathematical model there are essential nonlinearities, although here the quantities ν and a are taken to be constant and the equations are solved in the Boussinesq approximation, i.e., only ρ in β and the force A_β (a function of temperature) are assumed to be temperature-dependent.

For the nonstationary problem nonstationary terms with $\partial T/\partial \tau$ and $du/d\tau$ appear in Eqs. (23) and (24), i.e., a total derivative with $dT/d\tau$ and $du/d\tau$ appears in the differential equation.

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* Hydraulic resistances is the term commonly applied to forces, rather than to the coefficients at Δu.

Let us consider not only the groups R^{mech}, but also the relationships between these groups. We recall once again that comparatively simple expressions (without "account" for directions (!)) are obtained for the simplest form of the elementary (control) volume $\Delta V = h^3$, $\Delta S = h^2$, i.e., $h_{x1} = h_{x2} = \ldots = h_{z2} = h$ and $\Delta S_{xz} = \Delta S_{yz} = \Delta S_{xy} = \Delta S = h^2$, $\Delta T = T_i - T_\infty$ (i is the number of the node):

$$\frac{R^{mech}_{vyl}}{R^{mech}_{uxx}} = \frac{hu_x h^3}{\nu h^2 h} = \Delta \mathrm{Re}_x , \tag{30}$$

$$\frac{R^{mech}_{vy2}}{R_{uxy}} = \frac{hu_y h^3}{\nu h^2 h} = \Delta \mathrm{Re}_y . \tag{31}$$

Such a group can be considered in Eq. (28):

$$A_\beta R^{mech}_\nu = \frac{\beta g (T_i - T_\infty) \Delta V h}{\nu \Delta S} = \frac{\beta g (T_i - T_\infty) h^2}{\nu} = \Delta \mathrm{Gr}' . \tag{32}$$

As is seen, the group is "generated" by multiplying R^{mech} by the AE coefficient A_β.

The analytical solution presented, for example, in [25] gives more interesting information on the composition of the groups (i.e., on interrelations of subsystems *inside* the system-process under investigation). If we introduce the new variables

$$\Theta = \frac{T - T_\infty}{T_w - T_\infty} ; \quad \mathrm{Gr}_x = \frac{\beta g x^3 (T_w - T_\infty)}{\nu^2} ;$$

$$\mathrm{Pr} = \frac{\nu}{a} ; \quad z = \frac{y}{x} \left(\frac{1}{4} \mathrm{Gr}_x \right)^{1/4} ; \quad \psi = 4\nu \left(\frac{\mathrm{Gr}_x}{4} \right)^{1/4} \varphi (z) ;$$

$$v_x = \left[\frac{\beta g (T_w - T_\infty)}{4\nu^2} \right]^{1/4} 4\nu x^{1/2} \varphi' ,$$

$$v_y = \nu x^{-1/4} \left[\frac{\beta g (T_w - T_\infty)}{4\nu^2} \right]^{1/4} (z\varphi' - 3\varphi) , \tag{33}$$

then the mathematical model in partial derivatives transforms into an ordinary differential equation of the third order for φ and of the second order for Θ:

$$\varphi''' + 3\varphi\varphi'' - 2(\varphi')^2 + \Theta = 0 , \tag{34}$$

$$\Theta'' + 3\mathrm{Pr}\,\varphi\Theta' = 0 . \tag{35}$$

In this case boundary conditions (25) and (26) will take the form

$$z = 0 \quad \varphi = \varphi' = 0 , \quad \Theta = 1 , \tag{36}$$

$$z = \infty \quad \varphi' = 0 , \quad \Theta = 0 . \tag{37}$$

As we can see, this conversion reduces sharply the number of arguments, with Pr remaining the most important of these. This form of the groups indicates interdependences and relations *inside* the system. It is impossible to obtain the form of groups (33) by using methods of similarity theory [5-10]. This is one of the merits of analytical methods that is usually not noted by their experts.

The investigation of our thermal system is conducted in groups, i.e., the functions $\varphi'(z)$ and $\Theta(z)$ are constructed. Only the two arguments z and Pr remain. One other fact is of interest: $a = \lambda/(c\rho)$ is not just a characteristic group; it is part of the integrative criterion $\mathrm{Pr} = \nu/a$. At certain values of Pr (see [25], Fig. 3-46) we obtain systemic (!) integrative effect, namely, a maximum of the function $\varphi' = \nu_x[x/(2\nu)]\mathrm{Gr}_x^{1/2}$. This effect cannot be obtained without system analysis, i.e., without a solution of the problem stated and without a complex multilevel computational experiment. It should be emphasized that after all the transformations the thermal system, i.e., MM (34)-(37), was ultimately investigated numerically (!).

In the case of a nonstationary problem, we obtain a grid number $\Delta\mathrm{Fo}^{\mathrm{mech}}$ that involves $R^{\mathrm{mech}} = \delta\tau/\Delta V$:

$$\Delta\mathrm{Fo}^{\mathrm{mech}} = \frac{R_\tau^{\mathrm{mech}}}{R_\nu^{\mathrm{mech}}} = \delta\tau\nu/h^2, \quad \mathrm{Pr} = \frac{\mathrm{Fo}^{\mathrm{th}}}{\mathrm{Fo}^{\mathrm{mech}}} = \frac{a}{\nu}, \quad \Delta\mathrm{Pr} = \frac{R_\tau^{\mathrm{th}}}{R_\tau^{\mathrm{mech}}} = \frac{a_{\mathrm{loc}}}{\nu_{\mathrm{loc}}}. \tag{38}$$

The complexing of the groups is carried out by the method of self-similar variables described in [12, 13], which amounts to a reduction in the number of arguments to a minimum. The complexes remaining after transformations are the desired groups of subsystems.

The phenomenological theory of heat transfer makes use of similarity criteria, for example, $\mathrm{Re}_{\mathrm{cr}}$, $\mathrm{Ra}_{\mathrm{cr}}$, and many others. We will consider the criterion Re, which depends on the velocity u, the kinetic viscosity ν, and the determining dimension L. The integral number Re is interpreted as a measure of the relationship between inertia forces and forces of molecular friction. The local number $\Delta\mathrm{Re}$ represents the relationship between the mechanical resistances, i.e., the coefficients of the AE in the terms of the equations associated with pulse diffusion R_ν^{mech} and with the convective term R_{ux}^{mech} (see expressions (29)). The integral number $\mathrm{Re}_{\mathrm{cr1}}$ is equal to about 2300 for flow in tubes; $\mathrm{Re}_{\mathrm{cr2}} \approx 5 \cdot 10^5$ for flow around a plate; $\mathrm{Re}_{\mathrm{cr3}} \approx 100$ for isothermal flow around a sphere; $\mathrm{Re}_{\mathrm{cr4}} \approx 500$ for a cylinder in a transverse flow:

$$\mathrm{Re}_{\mathrm{cr1}} = \frac{UD}{\nu}; \quad \mathrm{Re}_{\mathrm{cr2}} = \frac{Ux}{\nu}; \quad \mathrm{Re}_{\mathrm{cr3(4)}} = \frac{UD}{\nu}. \tag{39}$$

As we can see, in all cases U is the inlet velocity, and ν is a coefficient depending on the kind of fluid. The determining dimensions D and x vary. It is evident that $\mathrm{Re}_{\mathrm{cr}}$ should involve all the geometric factors (form-factors); then the criterion of stability will have the form $K = f(U, \nu, a_j)$, where the a_j-type coefficients in each specific problem must reflect all the geometric factors, rather than one dimension x or D; the coefficients a_j must reflect all the interrelations and correlations between the flow and the outer and inner medium (a tube, an infinite volume – a ball, cylinder, sphere, etc.). A group is not simply a set of the subsystems U, L, and ν, but a set of all the subsystems that takes into account all the interrelations of the given specific system. In many publications the authors subject their results to a processing such that not simply is Re an argument, but also Re multiplied by coefficients that reflect the influence of a *complex* form of the object under investigation.

Different complexes of the type of Boltzmann substitution and integral transformations are forms of groups. This specific feature of integral transformations was noted by the authors of [5, 11]. Monograph [26] is devoted, in essence, to the search for forms of groups that reflect the structure of thermal systems, although the author himself does not write about interaction and emergence as basic properties of the "heat-transfer" systems.

NOTATION

$a = \lambda/(c\rho)$, thermal diffusivity; c, specific mass heat capacity; $c_V = c\rho$, specific volumetric heat capacity; g, acceleration of gravity; h, space interval, distance between nodes of a grid region; l, distance from a node at the center of ΔV to the surface of ΔV; L, determining dimension; n, normal; Q, heat flux (power); q, specific density

(power) of a heat flux; R, thermal resistance; S, area; T, temperature; u, velocity; V, volume; x, y, z, coordinates; $x_1 = x$, $x_2 = y$, $x_3 = z$; Bi, Biot number; Fo, Fourier number; Gr, Grashof number; Ki, Kirpichev number; M, Mach number; Nu, Nusselt number; Pe, Peclet number; Po, Pomerantsev number; Pr, Prandtl number; Ra, Rayleigh number; Re, Reynolds number; Sh, Strouhal number; α, coefficient of heat transfer; β, volumetric expansion of a liquid; ΔS, ΔV, elementary area, volume; ΔT, temperature difference in space, time; $\delta\tau$, time interval; μ, dynamic viscosity; Λ, thermal conductivity; λ, heat conduction; ν, kinematic viscosity; ρ, density; τ, time. Subscripts and superscripts: lat, lateral; liq, liquid; con, contact; cr, critical; loc, local; mech, mechanical; 0, characteristic; sur, surface; nom, nominal; med, medium; w, wall; th, thermal; V, volumetric; Σ, total. Abbreviations: AE, DE, ODE, algebraic, differential, and ordinary differential equation; DEPD, differential equation in partial derivatives; IP, integrative property; MM, mathematical model; SS, subsystem; S, system.

REFERENCES

1. L. A. Kolesnikov, Foundations of the Theory of System Analysis [in Russian], Kiev (1988).
2. P. Glensdorf and I. Prigogine, Thermodynamic Theory of the Structure of Stability and Fluctuations [Russian translation], Moscow (1973).
3. S. S. Kutateladze, Heat Transfer and Hydrodynamic Resistance: Handbook [in Russian], Moscow (1990).
4. D. Anderson, J. Tunnehill, and R. Fletcher, Computational Hydromechanics and Heat Transfer [Russian translation], Moscow (1990).
5. A. A. Gukhman, Trudy Mosk. Inst. Inzh. Zhel.-Dor. Transp., Issue 139, Moscow, 31-44 (1961).
6. A. A. Gukhman, Introduction to Similarity Theory [in Russian], Moscow (1973).
7. A. A. Gukhman, Application of Similarity Theory to Investigation of Heat Transfer Processes [in Russian], Moscow (1974).
8. S. S. Kutateladze, Analysis of Similarity in Thermal Physics [in Russian], Novosibirsk (1982).
9. I. I. Novikov and V. M. Borishanskii, Similarity Theory in Thermodynamics and Heat Transfer [in Russian], Moscow (1979).
10. J. Kuneš, Modelovani tepelných procesú, SNTL, Prague (1989).
11. A. V. Luikov, Tr. Mosk. Inst. Inzh. Zhel.-Dor. Transp., Issue 139, Moscow, 10-19 (1961).
12. A. A. Gukhman and A. A. Zaitsev, Teplofiz. Vys. Temp., 8, No. 1, 136-146 (1970).
13. A. A. Gukhman and A. A. Zaitsev, Teplofiz. Vys. Temp., 8, No. 4, 847-855 (1970).
14. L. A. Kozdoba, Computational Thermal Physics [in Russian], Kiev (1992).
15. O. I. Bukovskaya, L. A. Kozdoba, M. V. Kudinova, and P. G. Nagolkin, Prom. Teplotekhn., 17, Nos. 1-3, 70-79 (1995).
16. L. A. Kozdoba, V. P. Nevmerzhitskii, and S. V. Filin, Prom. Teplotekhn., 17, No. 4, 6-14 (1995).
17. L. A. Kozdoba, M. V. Kudinova, and A. P. Nagolkin, Prom. Teplotekhn., 17, No. 5, 35-41 (1995).
18. L. A. Kozdoba and M. V. Kudinova, in: Heat and Mass Transfer-MIF-96 [in Russian], Vol. 9, Pt. 4, Minsk (1996), pp. 106-111.
19. S. Patankar, Numerical Methods for Solving Problems of Heat Transfer and Fluid Dynamics [Russian translation], Moscow (1984).
20. A. P. Vanichev, Izv. Akad. Nauk SSSR, Otd. Tekh. Nauk, No. 12, 1767-1774 (1946).
21. L. A. Kozdoba, Electrical Simulation of Temperature Fields in Parts of Ship Power Plants [in Russian], Leningrad (1964).
22. L. A. Kozdoba, Electrical Simulation of Heat and Mass Transfer Phenomena [in Russian], Moscow (1972).
23. B. S. Petukhov, Heat Transfer in Moving Single-Phase Medium. Laminar Boundary Layer [in Russian], Moscow (1993).
24. P. Schneider, Engineering Problems of Heat Conduction [Russian translation], Moscow (1960).
25. A. V. Luikov, Heat and Mass Transfer. Handbook [in Russian], Moscow (1972).
26. A. G. Shashkov, System-Structural Analysis of Heat Transfer Processes and Its Applications [in Russian], Moscow (1983).